Intro

When you stand in front of the mirror, the image you see looks just like you. In mathematics, we call this a transformation—in this case, a reflection.

A transformation is a function that creates images which are congruent to the original figures. The original figure is called the **preimage** and the resulting figure the **image**. These types of transformations are known as **rigid transformations**.

There are three basic types of rigid transformations:

**Reflections**

**Rotations**

**Translations**

Transformation Notation

There are different types of notations for describing a transformation. One of the most common when working with a coordinate plane looks like this:

\[ P(x, y) \rightarrow (x + 3, y + 2) \]

So, putting it all together, the funky-looking thing above states: “The transformation of \( P \) that maps \((x, y)\) onto \((x + 3, y + 2)\).” This basically states to move the preimage 3 units along the \(x\)-axis and 2 units along the \(y\)-axis to obtain the image.

Let’s look at rigid transformations a little closer.
Translations

A **translation** “slides” a preimage onto an image. Also, a translation is a rigid transformation, both the preimage and image are congruent.

Remember the funky notation we used above? Let’s take a look at it once more.

\[ P(x,y) \rightarrow (x + 3, y + 2) \]

The transformation asks to move the preimage 3 units along the x-axis and 2 units along the y-axis. Here’s an example of how to use this transformation:

Our preimage (in blue) is a triangle.

The transformation calls for a *mapping* of \((x + 3, y + 2)\).

In other words, I need to move this triangle 3 unit along the x-axis and 2 units along the y-axis.

This movement creates my image (in purple).

This type of transformation is called a translation because we “slid” the preimage onto the image.

Let’s look at another example: \( T(x, y) \rightarrow (x + 5, y - 4) \)

Our preimage (in blue) is a triangle.

The transformation calls for a *mapping* of \((x+5, y-4)\).

In other words, I need to move this triangle 5 unit along the x-axis and -4 units along the y-axis (remember that negative numbers move down in the y-axis).

This movement creates my image (in purple).
Reflection

A **reflection** is the image of a figure that has been “flipped” over a line of reflection. A reflection is also a rigid transformation which means the preimage and image are congruent.

Reflections drawn on a coordinate plane are usually reflections along the $x$-axis, $y$-axis or the line through the origin at $45^\circ$ angle to the axes ($y = x$). What does this all mean?

Let’s take a look graphically:

Notice that in each case, the preimage is **always the same distance away** from the line of reflection as the image. Very important!

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4 Knowledge of Geometry

Apply concepts and properties of transformational geometry (e.g., dilation, translation, rotation and reflection),
From the examples above, we can conclude the following:

If \( P(x,y) \) is reflected over the \( x \)-axis, its image is \( P(x, -y) \).
If \( P(x,y) \) is reflected over the \( y \)-axis, its image is \( P(-x, y) \).
If \( P(x,y) \) is reflected over the line \( y = x \), its image is \( P(y, x) \).

This is useful to understand and keep in mind as you encounter questions which ask for figures to be reflected.

**Rotations**

A rotation is a transformation which is also called a “turn”. A rotation uses a point as its **center of rotation**. This center of rotation may or may not be on the figure. A rotation is a rigid transformation, which means the preimage and image are congruent.

When working with rotations, you need to include the following:

1. The amount of the turn (aka. The angle of rotation).
2. The center of rotation.
3. The direction of the turn (e.g. clockwise or counterclockwise).

Let’s look at some examples:

**FYI:** The negative sign means “the opposite of”. For example, -\( y \) means write the opposite of whatever the \( y \)-value happens to be.
Here is an example of a rotation (or turn) using a coordinate plane:

![Diagram of a rotation](image)

**Rotate the preimage 90 degrees clockwise about the center of rotation, \( R \).**

To rotate a figure on a coordinate plane:

If the point of rotation is not on the shape, draw lines connecting the vertices (corners) of your shape to the center of rotation. Remember, each corresponding point must be the same distance away on the image.

Next, rotate each vertex (corner), in this case 90 degrees clockwise, and draw your new vertices.

**Dilations**

There is another type of transformation, called a dilation, which you may encounter. A **dilation is not** a rigid transformation. A dilation enlarges or reduces the size of a shape; this is why the preimage and image of a dilation are not congruent, but similar.

Every dilation has a center point, \( C \) and a scale factor (or size change), \( k \).

![Examples of dilations](image)

- **The center point is \( C \).**
- **The scale factor, \( k \), is 0.5**
- **The dilation of this figure has made it half the original.**
- **The center point is \( C \).**
- **The scale factor, \( k \), is 1**
- **This is the original figure.**
- **The center point is \( C \).**
- **The scale factor, \( k \), is 1.5**
- **The dilation of this figure has made it one-and-a-half times larger than the original.**

Since dilations use a scale factor, \( k \), to determine their size: if the preimage has points \((x,y)\), then the image maps onto \((kx, ky)\) where \( k \) is the scale factor.

What does that mean? Well, let’s take a look at the following example:
What is the scale factor of ΔDEF to ΔPQR?

In order to solve this problem, first we have to determine if this dilation is a reduction or enlargement.

From the diagram on the left, we can see that the preimage, ΔDEF, is smaller than the image, ΔPQR. This tells us an enlargement took place.

To find the scale factor, simply find the length of DF and PR and the heights of each triangle.

DF is 4 units and PR is 12 units.
The height of ΔDEF is 2 and the height of ΔPQR is 6.

From this information, we can conclude the scale factor is 3 because ΔPQR is 3 times larger than ΔDEF. Using our notation from before, we can state:

\[ T(x,y) \rightarrow (3x, 3y) \]

Be careful!

If the question above asked for the scale factor of ΔPQR to ΔDEF, the answer would be \( \frac{1}{3} \). Also, the notation would change to: \( T(x,y) \rightarrow \left( \frac{x}{3}, \frac{y}{3} \right) \). Remember to read the order in which they state the question. This is a common mistake, so be careful!